AN ALTERNATING CURRENT RANGE AND POSITION FINDER.

An account of experiments at the U. S. Artillery School, Fort Monroe, Va.

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The desirability of having a range-finder using a horizontal base which may be a mile or more in length, is so generally admitted that it requires no arguments to be presented in its favor. The solution of the problem itself, namely, the finding of the ranges and positions, is one of the simplest operations of trigonometry after the angles are measured and the base is known. It is only the amount of time consumed in the process of first measuring the angles, and then making the calculations, which makes a range finder necessary at all. The chief requisite of a good range finder would seem to be quickness and reliability in action, and with that a certain necessary degree of accuracy.

The problem is however so well understood by the military world that it is a matter of common experience to meet those who have planned their own range finders, even if they have not been actually constructed. It is generally understood that the major part of the difficulty has been overcome when it becomes possible to cause an indicator of some kind to move in exact parallelism with the telescope of one of the base-end instruments a mile or more distant.

It is proposed in this paper to consider a new method, by which this problem of producing parallelism at a distance has been experimentally tried and it seems satisfactorily solved.

The perfection attained by Lieutenant Fiske in the use of the Wheatstone's bridge as applied in his Range Finder has attracted much attention, and many concede that they would not desire a better range finder than this principle affords if it were not for certain serious objections which develop only under the real test of actual practice. It is to these objections that it seems worth while giving one's careful attention and study; for if they are

removed, the result seems certain. Among some of the objections may be mentioned the number of adjustments necessary before operating, the difficulty which most operators would meet in repairing any part which might get out of order, the sliding electrical contacts, and the necessity for a low resistance line between the two distant stations.

In comparatively recent years the alternating current in electricity has received a tremendous commercial development, which has been at least an incentive to the thorough study of the various phenomena connected with it. These phenomena include many additional things over and above what one meets in direct current flow, and some of them have such an importance that the alternating current is finding its way continually to new practical applications for which the direct current is not adapted. In alternating current work the "inductance" of the line should be known as well as its resistance; the inductance giving rise to a "reactance" which in some ways has the same effect as a resistance. In fact this reactance may be and often is the factor which controls the current, and is much more important than the resistance of the circuit itself. This is one of the phenomena which usually surprises one trained in the old school of direct current electricity, where the ohm played the all important part.

It is the use of the alternating current with the Wheatstone's bridge to obtain parallelism, which is one of the essential differences in the method proposed from those already used. Nearly all the difficulties met with before seem to vanish naturally; the instrument need not be extremely delicate and difficult of adjustment, but may even be roughly handled without serious injury. The resistance of the wires is of little importance, and the line resistance need not be low; for example the iron telegraph wires already laid between the two stations where the base-end instruments are situated at Fort Monroe, Va., were used without alteration in these experiments, the distance being 1922.305 yards.

Perhaps the point most in its favor is the fact that all the wires may be permanently laid and encased, never to be touched, since there are to be no "sliding contacts" where a contact moves along a "perfectly uniform" wire, one whose length is proportional to its electrical resistance. The use of these sliding contacts is avoided in the present instance by causing a bundle of soft iron wires to move into and out from a solenoid. In this manner the reactance of the circuit is varied, and not the resistance. It is the reactance, and not the resistance, which plays the more

important part in governing the current, and here there is a means of varying the reactance without altering the electrical circuit at all, simply through the influence of the iron in the solenoid, which is transmitted through the ether.

METHOD OF EXPERIMENT.

In Fig. 1 is shown the ordinary arrangement of the circuits of a Wheatstone's Bridge supposed to contain in the branch numbered 6 a source of alternating E. M. F., and in the conjugate branch numbered 5 a mil-ammeter for detecting the presence of an alternating current. The branches 1, 2, 3, and 4 are supposed to contain coils with inductance. A disposition of these circuits, which makes it more evident how they may be used in an actual case is illustrated in Fig. 2, where the letters and numbers correspond. In this Figure the group of circuits and apparatus near Station H, is located at one of the observing stations, and that marked Station A at the other, while the generator may be at any convenient place, in the present instance at the Electrical Laboratory. A single line runs from the laboratory to the point *A*, Station H, which corresponds with circuit 6, Fig. 1. At *A* it divides along the branches 1 and 2, which are each stationary coils containing inductance, and arrives at the points *C* and *D*, between which points the galvanometer is inserted making branch 5. From *D* the circuit first passes around the coil which is to contain the movable iron core, and thence along the line to the point *B* at Station A, which is connected to the ground or the return. From *C*, the circuit passes immediately to the line running to Station A, and thence around the coil with the movable iron core and to *B*, the return.

The method of operation is as follows: The inductances of circuits 1 and 2 are fixed and constant, preferably approximately equal. If the iron core of circuit 3 is set in a certain position within the solenoid, then there is only one position of the core in 4, in one end of the coil, which will bring the galvanometer indication to zero. A motion of the cores is communicated to the telescopes or indicators by suspending the core vertically within its coil by means of a steel ribbon, which passes up over a pulley having a horizontal axis, and thence around a large circle with a vertical axis upon which the indicator is mounted. In practice the core is moved by turning the large circle.

If the coils 1 and 2 with their cores are made exactly alike and also the coils and cores of 3 and 4, it is evident that the positions of the cores 3 and 4 will always be the same for zero galvanom-

eter current. Hence if the large horizontal wheels are alike, the telescopes or indicators will always remain parallel for a zero galvanometer reading. The objection may be raised that it is impossible to make the coils and cores exactly alike, which must be admitted. A further consideration however will show that this condition of equality is not a necessary one, though desirable; for even though the electrical parts are exactly alike, the motion of the horizontal circles may be altered at pleasure by slightly changing the shape of the wheel from a true circle, or by making a cam of it. This plan is not proposed, however, as it is found in practice that the motions are alike within the error of setting by the galvanometer, even though the coils are only approximately alike. This cause in other words does not seem to be the largest source of error.

APPARATUS USED.

The preliminary experiments were made in the laboratory of the U. S. Artillery School during July and August, 1896, to determine first roughly whether the plan would succeed. This apparatus is shown in the view Fig. 3, where may be seen resting upon a high table two sets of instruments, practically alike. The coil AC (1) corresponding to circuit (1) Fig. 1 rests upon the table and has a bundle of iron wires for its core. The coil AD (2) is a similar coil corresponding to circuit 2. CB (3) is a coil screwed to a board in the vertical position, having an iron core I attached to a flexible steel ribbon R which passes over the pulley P and thence around the horizontal wheel T. This coil corresponds to circuit (3) Fig. 1 and the similar vertical coil DB (4) to circuit (4).

THE GALVANOMETER.

The instrument needed as a galvanometer to be used for zero readings must be one suited to indicate alternating currents of small magnitude. These instruments are not among those which are manufactured for commercial purposes at present, so that resort was made to the nearest thing available. A telephone was tried and worked fairly well, but the ear does not seem to be so good an organ to use as the eye, and finally a universal galvanometer adapted to serve as a Siemens' dynamometer gave much better results. When a range finder is constructed, a special instrument will be made adapted to the purpose, which will probably increase the accuracy of the results.

The deflection of the coil of the dynamometer with alternating currents is always towards one side, so that in practice the needle

is brought down to its minimum reading instead of a zero reading. This minimum is not always the zero reading, though very near to it; for the current never falls exactly to zero unless the wave of current flow is exactly harmonic.

METHOD OF TESTING FOR PARALLELISM.

An optical method was employed in the laboratory in making the test for parallelism, similar to that used in the well known Sextant instrument. Two plane mirrors M and M', Fig. 3, were mounted one on the rotating table of each instrument. The diagram Fig. 4 illustrates the mode of operation. The observer at E looks directly at a distant object O through the laboratory window, and sees the same object by reflection in the mirrors M and M', then brings the objects into coincidence by rotating one of the mirrors. The mirrors are parallel when the objects coinside, and in this position they are clamped while the galvanometer is standing at zero. After this adjustment is made one of the horizontal wheels is turned together with its mirror, which operation moves the iron core either up or down. This operation throws the galvanometer out of position, and it will not return until the other wheel and mirror with its iron core are moved a corresponding amount. The galvanometer then again indicates zero, and it is also found that the object and the image seen in the mirrors are again brought nearly into coincidence.

In this trial the optical test was more severe and accurate than the galvanometer indication, but the coincidence was so close and parallelism so nearly attained that it was decided to give the method a more thorough test by instruments where these small differences, the errors, could be measured. A motion of the iron core through a distance of one one-hundredth of an inch made a perceptible difference in the galvanometer indication.

An inspection of the base-end instruments already in use at Fort Monroe, suggested that the apparatus already made might be used in conjunction with those instruments as a kind of attachment without making any alteration whatever. Fortunately this was made possible because there happened to be a slight elevation of the rim of the circle of the base-end telescope, just high enough to admit of winding around it the steel ribbon already mentioned. This rim is a circle 16 inches in diameter, and revolves in a circle of the same size made to receive it. The bearing circle is imbedded in cement, and supported upon a brick pier built up from the sand of the beach.

In Fig. 5 is seen a view of this base-end instrument situated at the station known as "Station H." This station is a small

wooden structure situated at the water's edge upon the sandy beach. The brick pier is built up from the sand, and one of the instruments shown in Fig. 3 is seen here fastened to the side of the pier by the iron bolts *BB*. The coil *DB* (4) and core *I'* with pully *P'* having the ribbon *R'* passing around it are the same as those seen in Fig. 3. The ribbon then passes directly around the 16-inch circle and is clamped to it. The telescope carries with it upon an arm the vernier, which moves inside the rim of the graduated circle. The circle itself revolves in the outer bearing circle already mentioned, and either telescope or circle may be clamped to its bearing.

Fig. 6 shows a view of the other instrument which was attached in a similar manner to the pier at the station known as "Station A," also situated upon the beach at the water's edge; the distance between the two stations being 1922.305 yards, or 1757.6 meters.

The wiring is shown in Fig. 2 where the alternator *M* is in the laboratory, from which a single wire runs to Station H, a distance of 810 yards. Two wires run from Station H to Station A, a distance of 1922.305 yards, and a ground return is used for both.

Station A is 1868 yards from the laboratory. It will be seen that it is not necessary to have two lines run between Stations H and A; for the line marked *DB* (4) simply goes from H to A to be grounded. It may as well be so grounded at Station H, and the reason for using it in the present instance was merely the practical one, that if it were not used a substitute coil would have had to be made which would exactly simulate the line, as the two lines 3 and 4 should have the same impedance. The easiest way to accomplish this was to use the two lines already laid.

THE MANNER OF USING THE BASE-END INSTRUMENTS.

The only vernier upon the instrument is attached to the telescope and revolves with it upon the inside of the graduated circle. It became necessary, in using the instrument as intended, to provide some way of reading the angle through which the outside circle turns. As ordinarily used this outside circle remains at rest, and the telescope alone with the vernier moves, but in the present instance the large circle is required to move in order to wind up the steel ribbon.

To overcome this difficulty the following plan was adopted by which the inside vernier became available.

Some distant object is selected by each observer, not neces-

sarily the same object for both, though as a matter of fact the same object happened to be used on account of its convenience. It was the pole on the top of "Bug" light, a light house 6500 yds. from Station H. While the telescopes are kept set upon this object, the coils are raised or lowered in their solenoids by revolving the outer circle without the telescope, until the galvanometer indicates a minimum. In this position the telescopes are clamped, that is, while they are each pointing towards Bug Light, and the galvanometer stands at a minimum. The reading of the vernier upon its circle is then recorded. The circle is divided into half degrees, and the vernier reads to minutes. Next, the telescope at station A is turned without its circle through any desired angle, say ten degrees, for example, and clamped again. The telescope and circle with the iron core are then revolved together until the telescope again points to Bug Light. By this method the large circle can be revolved through any given angle by simply reading the vernier already provided. The second observer next moves his telescope with circle and iron core until the galvanometer indicates a minimum, and clamps the circle. The telescope is then unclamped and turned back to Bug light, and the reading of the vernier recorded. If the instrument works without error this angle should be the same as that of the first instrument, namely ten degrees. The differences in the readings of the two instruments indicate the errors.

The range through which the telescopes could turn was about 125 degrees, and was limited only by the length of coil and iron core, which is a matter that may always be arranged so as to admit of as wide a field as desirable. In these trials it was found that the galvanometer would indicate when the telescope had moved over the same angles within the error of setting. Inasmuch as the other errors are smaller than the error of setting, therefore it will only be necessary to give the error of setting. By the error of setting is meant the error which is made by the observer at station H in trying to set his telescope corresponding to that at station A by means of the galvanometer observations alone. An example giving the readings for twenty independent settings made by the observer at station H for one given setting of the telescope at station A will make this more clear. The vernier at station A was set to read exactly 180 degrees, and twenty observations were made at station H to set the telescope by the galvanometer indication, moving the telescope out of adjustment each time to make sure of independent settings, according to the following table :

No.	Reading.	Difference from the mean.	Difference squared.	
No. 1	14° 49′	6.75	45.5625	
2	" 39	3.25	10.5625	
3	" 46	3.75	14.0625	
4	" 40	2.25	5.0625	
5	" 38	4.25	18.0625	
6	" 42	.25	.0625	
7	" 42	.25	.0625	
8	" 50	7.75	60.0625	
9	" 37	5.25	27.5625	
10	" 37	5.25	27.5625	
11	" 39	3.25	10.5625	
12	" 41	1.25	1.5625	
13	" 42	.25	.0625	
14	" 41	1.25	1.5625	
15	" 42	.25	.0625	
16	" 42	.25	.0625	
17	" 42	.25	.0625	
18	" 43	.75	.5625	
19	" 42	.25	.0625	
20	" 51	8.75	76.5625	
Mean	14° 42′.25	2′.775	299.7500 =	$\Sigma e^2 = s$

Mean $e = 2'.775$

Mean value $= 14° 42.'25$

$$\text{Probable error of result} = .674 \sqrt{\frac{s}{n(n-1)}} = 0'.598$$

$$\text{Probable error of single observation} = .674 \sqrt{\frac{s}{n-1}} = 2.'67.$$

It is seen by the table that the mean error for twenty observations is 2.775 minutes of arc and the greatest error is 8.75 minutes. Remembering that a minute is about the angle subtended by one foot and a half, or half a yard at a distance of a mile, this mean error of 2.775 minutes is equivalent to 1.39 yards at the distance of a mile, or 7.9 yards at the distance of 10,000 yards, which is at present about the limit of firing distance, being equal to 5.68 miles. An instrument which will locate a target within 8 yards at a distance of 10,000 yards comes within the limits of requirement for a good range finder. The probable error of a single observation is still smaller than the mean, being only 2.67 minutes. Perhaps the greatest error is of more importance than the mean, for when the greatest error is well within the required limits, the location of the target is more certain. In the present instance the greatest error of 8.75 minutes corresponds to about 25 yards in 10,000.

The erroneous conclusion must not be drawn from the above that this is the limit of accuracy attainable with this form of instrument. The above result exceeded the experimenters' expectations as to accuracy, it being understood that whatever accuracy was attained by this instrument might be increased with certainty three or four fold. For example the diameter of the large circle was only 16 inches, and by doubling this, the accuracy is immediately increased two-fold. It will be noted that only one size and length of coils and cores have yet been tried, being those which were first constructed for the purpose, in combination with base-end instruments not constructed with direct reference to this object. The error of setting is not the same for all positions of the iron core, being less when it approaches the center of the coil than it is further out, and less again when it is too far out, so that there is a point of maximum sensibility. The range through which the indication is sufficienly sensitive is however long enough to cover the desired field.

DISCUSSION OF THE CURRENTS IN THE BRANCHES OF A WHEATSTONE'S BRIDGE, WHERE EACH BRANCH CONTAINS RESISTANCE AND INDUCTANCE, AND THERE IS AN HARMONIC IMPRESSED ELECTROMOTIVE FORCE.

The complete discussion of the currents flowing in the branches of a Wheatstone's Bridge, with a simple harmonic E.M.F., leads by the analytical method to equations which are too cumbersome to be of much practical use. For instance, by the direct method there would be formed as many differential equations as there are branches to the bridge, namely six, and these six simultaneous equations would by a process of elimination lead to a single differential equation of the sixth order, the solution of which would give the current in any desired branch. The practical obstacles in the way of finding the single differential equation from the six simultaneous equations, and after that obtaining its solution, are so great that if the same results can be obtained in a simpler way it would be welcomed. Having once written down the equations of current flow in the branches, it is a comparatively easy matter to determine the conditions for zero current in the galvanometer.

It happens that for this important particular case of zero current in the galvanometer, there is a very simple way of determining these conditions for an harmonic E.M.F. Either the analytical or the graphical method will give the result, but the

graphical method apparently gives so much more at the same time that it is preferred. The use of these graphical methods is so general and becoming now so well understood, that it is not considered desirable to repeat here the propositions which logically precede a complete understanding of the present discussion. A reference is made for those who desire to pursue the subject more thoroughly, to the graphical treatment* of Alternating Currents.

Referring to Fig. 1, let the six branches of the bridge be denoted by the numbers 1 to 6, and denote the resistances and inductances of the branches by the letters

$$R_1, R_2, R_3, R_4, R_5, R_6, \text{ for resistances}$$

and

$$L_1, L_2, L_3, L_4, L_5, L_6, \text{ for inductances.}$$

Let n denote the number of complete periods per second of the E.M.F. of the generator, and let T be the time of one period. Then $n = 1/T$. If a point move uniformly around the circumference of a circle, then the projection of that point upon any diameter has simple harmonic motion, and the angular velocity ω of the point on the circumference is $2\pi/T$ or $2\pi n$.

In Fig. 7, let the line OA represent the impressed E.M.F., E, at the terminals AB, Fig. 1, of the Wheatstone's Bridge. Since in the present discussion no current flows in the galvanometer circuit, then no change will take place in the other currents which are flowing, if the galvanometer circuit is broken. Imagine it to be so broken, and we have merely the case of a divided circuit of two branches, each branch containing two coils with impedance. To find the currents in each branch independently, a rule analogous to Ohm's law is applied. This states that the current I is equal to the E. M. F. divided by the impedance, or,

$$(1) \qquad I_{1,3} = \frac{E}{\Sigma J} = \frac{E}{\sqrt{(\Sigma R)^2 + (\Sigma L\,\omega)^2}}.$$

The angle by which this current lags behind the impressed E.M.F., OA, is given by the equation;

$$(2) \qquad \tan\theta_{1,3} = \frac{\Sigma_{1,3} L\,\omega}{\Sigma_{1,3} R}.$$

The subscript 1,3 refers to the current in the branches (1) and (3).

In Fig. 7, make the angle AOC equal to arc tan $\theta_{1,3}$ of equation (2), and this determines the direction of the current $I_{1,3}$. Then upon the line so drawn take the point D so that OD represents

* Bedell and Crehore on "Alternating Currents" Part II, W. J. Johnston Company, New York. Bedell on the "Principles of the Transformer." Mac Millan & Company.

in length the number of units contained in $I_{1.3}$ equation (1). The line OD then represents the current flowing in the circuits (1) and (3). A similar calculation may be made for the branch (2) and (4), and another current OD' drawn upon the diagram. Having determined the two branch currents OD and OD', the current in the main line is the geometrical sum of these currents, represented by the line OR.

The potential difference between the points C and D of the bridge is by hypothesis zero, and that between the terminals A and B is E. It remains to find the potential difference between the points A and C, A and D, C and B, and D and B. Draw a perpendicular from A upon the line OD, then OC represents the whole of that component of the total E.M.F., E, which goes to overcome the resistance of the two circuits (1) and (3). This E.M.F. following Ohm's law is equal to the product of the current and resistance. Hence $OC = (R_1 + R_3)\, I_{1.3}$. Similarly a point C' may be found upon the line OD', and $OC' = (R_2 + R_4)\, I_{2.4}$. These lines OC and OC' may then be divided into two parts proportional to the resistances R_1, R_3 and R_2, R_4. Such a division gives the points H on OC, and K on OC', so that $OH = R_1\, I_{1.3}$; $HC = R_3\, I_{1.3}$; $OK = R_2\, I_{2.4}$ and $KC' = R_4\, I_{2.4}$.

Just as OC represents the component of the total E. M. F., E, which is in the direction of the current, and may be called the "power E.M.F.," so CA represents that component which is at right angles to the current, and is called the "reactive E.M.F." This part of the total E.M.F. is due to the presence of the magnetic field in the circuits, and it may be divided up in a similar manner into parts which represent the separate effects of the coils (1), (2), (3) and (4). The line CA is equal to $(\Sigma_{1.3}\, L\omega)\, I_{1.3}$ and may be divided at M into the two parts $CM = L_1\, \omega\, I_{1.3}$ and $MA = L_3\, \omega\, I_{1.3}$. Similarly $C'A$ is divided at N into two parts $C'N = L_2\, \omega\, I_{2.4}$ and $NA = L_4\, \omega\, I_{2.4}$.

To find the potential difference at the terminals of any coil as AC, Fig. 1, we need only combine the componont E. M. F's. for that coil, one in the direction of the current and the other at right angles to it. This gives for circuit AC (1) the line $OH = R_1\, I_{1.3}$ in the direction of the current, and $HP = L_1\, \omega\, I_{1.3}$ at right angles to the current, to be geometrically added together, making the resultant OP the potential difference at the terminals of the coil AC (1). A similar process gives OQ for circuit AD (2), and PA for CB (3), and QA for DB (4).

Fig. 7 is drawn to represent the case of a divided circuit, and by joining a circuit across between the points C and D becomes

the Wheatstone's Bridge. If such a connection were made, the arrangement of the lines as drawn would be disturbed by the current which flows across the new circuit. Before the connection is made however, it is evident that the potential difference is represented by the line PQ in the Figure; for the closed circuit ACD Fig. 1 contains no impressed E.M.F. Hence the total fall of potential around the circuit must be equal to zero. The three E.M.F's in these branches must therefore form a triangle, and as the E.M.F's. OP and OQ are determined, it follows that the E M.F. PQ must be that between the terminals CD (5).

We then have:

The condition for no current in the galvanometer is that the points P and Q in the diagram shall coincide, or that the line PQ shall be zero.

In Fig. 8 is represented another diagram constructed like Fig. 7, except that the points P and Q are brought into coincidence at P. The condition that P and Q shall coincide, by referring to the Figure, and observing that the points H and K lie upon a semi-circle having OP as diameter, while M and N lie upon one having PA as diameter, may be expressed by the equations derived from the right triangles OHP and OKP, viz:

$$OP^2 = OH^2 + HP^2 = OK^2 + KP^2$$

(3) or $$I_{1,3}^2 (R_1^2 + L_1^2 \omega^2) = I_{2,4}^2 (R_2^2 + L_2^2 \omega^2).$$

Again

$$PA^2 = PM^2 + MA^2 = PN^2 + NA^2$$

(4) or $$I_{1,3}^2 (R_3^2 + L_3^2 \omega^2) = I_{2,4}^2 (R_4^2 + L_4^2 \omega^2).$$

Dividing equation (3) by (4), member for member, we obtain

(5) $$\frac{R_1^2 + L_1^2 \omega^2}{R_3^2 + L_3^2 \omega^2} = \frac{R_2^2 + L_2^2 \omega^2}{R_4^2 + L_4^2 \omega^2},$$

or denoting the impedances of the branches by J_1, J_2, J_3 and J_4, respectively, and remembering that $J_1 = \sqrt{R_1^2 + L_1^2 \omega^2}$; $J_2 = $ &c., we have

(6) $$\frac{J_1}{J_3} = \frac{J_2}{J_4},$$

as the necessary condition for zero current in the galvanometer.

This equation asserts:

When an harmonic electromotive force is impressed upon one of the branches of a Wheatstone's bridge, a galvanometer in the conjugate branch of the bridge can only indicate zero current when the impedances of the remaining four branches of the bridge form a simple proportion.

This is entirely analogous to the well known condition when the direct current is used in the Wheatstone's Bridge, namely, that

the resistances in the branches of the bridge shall form a proportion.

Another expression which the diagram makes apparent, and will be useful to note, is that derived from the triangles $O A C$ and OAC' inscribed in the semi-circle. These give

$$OA^2 = OC^2 + CA^2 = OC'^2 + C'A^2,$$

or

$$E^2 = (R_1 I_{1,3} + R_3 I_{1,3})^2 + (L_1 \omega I_{1,3} + L_3 \omega I_{1,3})^2 = (R_2 I_{2,4} + R_4 I_{2,4})^2 + (L_2 \omega I_{2,4} + L_4 \omega I_{2,4})^2,$$

(7)
$$E^2 = I^2_{1,3}\left[(R_1 + R_3)^2 + (L_1 + L_3)^2 \omega^2\right] = I^2_{2,4}\left[(R_2 + R_4)^2 + (L_2 + L_4)^2 \omega^2\right].$$

Adding (3) and (4), we have

(8)
$$I^2_{1,3}\left[R_1^2 + L_1^2 \omega^2 + R_3^2 + L_3^2 \omega^2\right] = I^2_{2,4}\left[R_2^2 + L_2^2 \omega^2 + R_4^2 + L_4^2 \omega^2\right].$$

Subtracting (8) from (7), we have

(9)
$$I^2_{1,3}\left[R_1 R_3 + L_1 L_3 \omega^2\right] = I^2_{2,4}\left[R_2 R_4 + L_2 L_4 \omega^2\right].$$

This may be written so as to express the ratio of the currents in the branches, as

(10)
$$\frac{I^2_{1,3}}{I^2_{2,4}} = \frac{R_2 R_4 + L_2 L_4 \omega^2}{R_1 R_3 + L_1 L_3 \omega^2}.$$

Equations (3) and (4) may also be so written as to express this ratio, and we have

(11)
$$\frac{I^2_{1,3}}{I^2_{2,4}} = \frac{R_2^2 + L_2^2 \omega^2}{R_1^2 + L_1^2 \omega^2} = \frac{R^2 + L_4^2 \omega^2}{\lambda_3 + L_3^2 \omega^2} = \frac{R_2 R_4 + L_2 L_4 \omega^2}{R_1 R_3 + L_1 L_3 \omega^2} = \frac{J_2^2}{J_1} = \frac{J_4^2}{J_3^2}.$$

In regard to the problem before us, namely, the application of this Wheatstone's Bridge to the range-finder, the resistances R_1, R_2, R_3, and R_4 as used are each permanently fixed in position once for all never to be moved. This meets any objection to the use of sliding contacts over uniform wires, and moreover the resistances are made small as compared with the inductances so that any variation which might happen from any cause, such as temperature, would have so much the less effect. The balance of the Bridge as used is rather between the reactances than the

resistances. Moreover the inductances of the two branches AC (1) and AD (2) Figs. 1, 2, and 3 were also made constant, so that of the entire eight quantities R_1, R_2, R_3, R_4, L_1, L_2, L_3 and L_4, six of them are constant and only two, namely L_3 and L_4, are variable.

It is possible therefore to calculate from the preceding conditions the value of L_3 for every value of L_4 which will produce zero current; for example if we denote the reactance $L_3\,\omega$ by x and the reactance $L_4\,\omega$ by y, then equation (11) may be written in the form

$$(12) \qquad k = \frac{m + y^2}{n + x^2} = \frac{p + q\,y}{r + s\,x},$$

where k is a constant equal to $\frac{I^2_{1,3}}{I^2_{2,4}} = \frac{R_2^2 + L_2^2\,\omega^2}{R_1^2 + L_1^2\,\omega_2}$, and $m = R_4^2$; $n = R_3^2$; $p = R_2\,R_4$; $q = L_2\,\omega$; $r = R_1\,R_3$; and $s = L_1\,\omega$.

Equation (12) may be reduced to

$$(13) \qquad qx^2y - sxy^2 + px^2 - ry^2 + ex + fy + g = 0,$$

where

$$e = -\,ms = -\,R_4^2\,L_1\,\omega\,; \; f = qn = R_3^2\,L_2\,\omega\,;$$

and

$$g = pn - mr = R_2\,R_4\,R_3^2 - R_1\,R_3\,R_4^2.$$

This is evidently an equation of the third degree between two variables, the reactances of the variable coils at the two stations in the range finder problem; and it enables one to find that value of the inductance at the distant station, which will just balance any inductance at the other station. This equation is of use in calculating points of maximum sensitiveness, and it also shows other useful relations. For instance, in the present case the resistances R_1 and R_2 are made approximately equal as well as the inductances L_1 and L_2. The equation shows that not only is it necessary for the inductances of the other two branches (3) and (4) to be equal for zero current, but the resistances of those branches must also be equal. Substituting $x = y$ in equation (13) satisfies the equation if $R_1 = R_2$; $R_3 = R_4$; and $L_3 = L_4$.

This point may be clearly seen in the diagrams. In Fig. 8, in which the points P and Q of Fig. 7 were merely brought into coincidence, it will be seen that

$$(14) \qquad \tan POH = \frac{L_1\omega}{R_1},$$

$$(15) \qquad \tan POK = \frac{L_2\omega}{R_2},$$

(16) $$\tan APM = \frac{L_3\omega}{R_3},$$

(17) $$\tan APN = \frac{L_4\omega}{R_4}.$$

By making $R_1 = R_2$ and $L_1 = L_2$ we reduce (14) and (15) to equality, so that, as the angles are the same, the points H and K are brought into coincidence at H, Fig. 9, upon their circle $OHKP$. Since the points C and C', Fig. 8, lie in the lines OHC and OKC' as well as upon the circle $OCC'A$ they are also brought into coincidence at C, Fig. 9. As the points M and N lie both upon the lines AMC and ANC and the circle $ANMP$, these points are also brought into coincidence at M, Fig. 9. By equation (11) the branch currents OD and OD' are made equal by these conditions, so that the points D and D' coincide in the figure, while $OR = 2\,OD$, and has the same direction.

VARIATION DIAGRAMS.

In Fig. 9 are represented the currents and potentials for a single value of the resistances and inductances only. If different inductances L_1 and L_2 are used, while all the other qualities remain the same, the diagram would be completely altered, and it is very desirable to know how the various points in the diagram move with a gradual change in the two inductances, provided always the galvanometer remains at zero. Fig. 10 illustrates this variation, and shows that the point P moves along the circle $AZPY$ in the direction of the arrow as the inductances L_1 and L_2 increase. The point M moves upon the circle XMA in the direction of the arrow as the inductances increase. The points H, C, and D move along the circles ZHO, ACO, and DO as the inductances increase. This diagram enables one to draw the true diagram for any value of the inductances. The points do not move on the dotted parts of the circles, but are limited to the heavy portions. The diameter AX is at right angles to OA, and the line PM produced passes through the point X. The triangle APM is similar to itself in all positions, for the angle at P must be constant and equal to arc tan $L_3\omega/R_3$. Hence the diameter AY makes an angle XAY with AX, equal to MAP. The point C always lies on the semi-circumference OCA. Hence the points H and D move on similar circles, their distances from O always being proportional to OC.

THE CONSTANTS OF THE CIRCUITS AS USED IN THE EXPERIMENTS.

Four coils were wound nearly alike upon four similar spools.

These spools were 44 cms. long, and 2.5 cms. in diameter to the windings. The hole through the center is 2.3 cms. in diameter. The wire is No. 22 single cotton insulated copper wire wound in 8 layers of 492 turns per layer making a total of 3,936 turns. The four iron cores were all made alike of soft iron wire, No. 16 insulated with single cotton, and cut into uniform lengths of 44 cms. The mass of each core was 407 grams. After the coils were made their resistances were measured by the Wheatstone's bridge to be,

Coil *AC* (1) . . . . . . 16.11 ohms.
Coil *AD* (2) . . . . . . 16.28 ohms.
Coil *CB* (3) . . . . . . 16.20 ohms.
Coil *DB* (4) . . . . . . 16.17 ohms.

The reactances and inductances were also measured for each coil, both when the iron cores are removed for minimum reactance, and when they are in the central position, for maximum reactance. These measurements were taken by the voltmeter and ammeter. Writing the equation for current in an alternating circuit, we have,

$$I=\frac{E}{J}=\frac{E}{\sqrt{R^2+L^2\omega^2}}.$$

Solving the above for J and for $L\omega$ we may write,

$$J=\frac{E}{I},$$

$$L\omega=\sqrt{\frac{E^2}{I^2}-R^2}=\sqrt{J^2-R^2},$$

where the quantities that occur in the right hand members are easily measured. For example the voltmeter reading at the terminals of coil AC (1) was 95.5 volts, while the ammeter in circuit read 4.40 amperes. Hence

$$J_1=95.5/4.40=21.6 \text{ ohms (impedance)},$$

$$L_1\omega=\sqrt{21.6^2-16.11^2}\doteq 14.5 \text{ ohms (reactance)}.$$

and knowing the speed of the generator to be such as to make ω equal to 876, we find that

$$L_1=.0166 \text{ henrys (inductance)}.$$

This was the value obtained when no iron core is inserted. When the iron core is inserted, the current falls off rapidly, so much so that the ammeter used before would no longer answer the purpose. In this case the current was determined by reading

the potential difference at the terminals of a non-inductive resistance inserted in the circuit, the resistance being previously determined.

Incandescent lamps were used cold, their resistance having been so measured, as the non-inductive resistance. Its value was 213 ohms. The voltmeter read 18.75 volts at its terminals, giving a current,

$$I_1 = \frac{18.75}{213} = .089 \text{ amperes.}$$

The reading at the terminals of coil (1) was 82.5 volts. This gives the impedance

$$J_1 = \frac{E_1}{I_1} = \frac{82.5}{.089} = 926 \text{ ohms.}$$

It is seen how the insertion of the iron core has increased the impedance, mainly on account of the reactance of the circuit. In fact the resistance, although 16.11 ohms, is so small that it has no appreciable effect upon the impedance and so upon the current; for example, the reactance

$$L_1\omega = \sqrt{J_1^2 - R_1^2} = \sqrt{926^2 - 16.11^2} = 925.85, \text{ only .15 of an ohm less than the whole impedance above.}$$

The value of ω obtained from the speed of the generator at the time was 853, hence

$$L_1 = 1.084 \text{ henrys (inductance).}$$

The values of the inductances of the four coils with and without the iron cores were found to be alike within the error of measurement. The value of the inductance of each coil is not far from 1.08 henrys with iron cores, and .017 without iron cores.

It is interesting to note that the inductance is increased about 64 fold by inserting the iron core.

In Fig. 11 is a diagram representing currents and E.M.Fs. as actually used. The impressed E.M.F. was obtained from the secondaries of five transformers connected in series in the laboratory, which gave about 300 volts at the terminals of the bridge. This is represented by the line *OA*. The resistances of the circuits *AC* and *CB* being 16.11 and 16.20 ohms respectively give a total of 32.31 ohms in one branch. The reactances of the same circuits are about 14.5 ohms each with no iron, and 926 ohms with iron. This makes the impedance of two circuits in series with no iron,

$$43.4 \text{ ohms,}$$

with iron in both,

1852 ohms,

with iron in one,

940.35 ohms.

The current OD equal to I, is therefore, with no iron,

$$I = \frac{300}{43.4} = 6.9 \text{ amperes,}$$

with iron in both,

$$I = \frac{300}{1852} = .162 \text{ amperes,}$$

with iron in one,

$$I = \frac{300}{940.35} = .318 \text{ amperes.}$$

Fig. 11 represents the case when no iron is in either coil, and shows the variation of the points as two cores are inserted one into each coil, until they are entirely in. The angle AOC is laid off for no iron, equal to

$$\text{arc tan} \frac{\Sigma L\omega}{\Sigma R} = \frac{29}{32.31} = .9 \text{ approx.}$$

Hence $\theta = 42°$ approximately. The E.M.F. OC is therefore 222.9 volts, and as the resistances are approximately equal, the point H (the letters of all the Figures denoting corresponding points) bisects OC. As the reactances of all coils are approximately equal, the point M also bisects AC, and hence P lies at the middle point of OA. The current OD is 6.9 amperes, in each circuit, making a total of 13.8 amperes in the main line. Any change made by introducing the two iron cores, has the effect of moving the points upon the heavy portions of their respective circles in the directions of the arrows as the cores are introduced, until the point P arrives at P', when the cores are entirely in. The points H, C, M, and D move to H', C', M', and D' when the iron cores are inserted into one pair of coils.

CONCLUSION.

During the progress of the experiments described above, a number of possible variations which naturally suggested themselves were tried, the details of which need not be given at this time. The influence upon the error of setting by means of the galvanometer, produced by placing the two fixed coils of the bridge at station H so that they would have a mutual induction between them was tested, as well as the effect of arbitrarily inserting inductance in series with the iron telegraph line wire running

between the stations. To secure a mutual induction between the fixed coils of the bridge, it was only necessary to place them side by side and then insert insulated iron wires bent around to form a continuous ring through both coils. This enabled the mutual induction to be increased gradually by adding wires, and the effect upon the galvanometer observed at each stage. By reversing the connections of one of the coils thus mutually related so that the magnetomotive forces of the two coils were opposed to each other, this case was easily tested. In general it may be said that these variations in the arrangement of the bridge gave results inferior to those outlined above. The effect upon the error of setting of decreasing the inductance of the fixed coils by wholly or partly removing the iron cores from them was noted, and as expected, produced a decreased sensibility.

Although much has been accomplished during the last ten years towards developing a range and position finder suited to the actual needs of sea-coast forts, yet the subject still seems far from settled, which is the more unfortunate since upon the range finder system adopted depends in a great measure the entire detail of fire direction and control.

Any discussion of the merits of horizontal base *versus* vertical base instruments is clearly uncalled for, but other things being equal, the desirability of having a base which may be a mile or more in length, operated from stations which may be effectively screened and protected from the enemy's fire, seems apparent.

One of the greatest arguments which has been urged against the horizontal base system, is the difficulty which two observers experience in turning their telescopes upon the same object from the two ends of the base. Mistakes are often made and much time is required to correct them by telegraph or telephone.

The above objection to the use of a horizontal base, does not seem necessarily inherent with the system, as a number of remedies may possibly be found. For instance, it is possible as has been suggested, that from comparatively rough observations for range and azimuth from the principal base-end. relocated by a simple attachment, that the approximate azimuth of the object selected can be sent at once to the other observer so that when he is directed to set his telescope at a certain azimuth, if the object is not found within the field of view of his telescope, it will at least be near enough in line to enable it to be readily identified.

From the very nature of the problems of sea-coast fire, a position finder should include some form of accurate map or plotting

board by which the positions of the moving object can be located as well as its future positions predicted. After so much care has been devoted to accurately measuring the angles and producing parallelism at a distance, this should not be vitiated by a rough and poorly designed method of mechanical plotting, in other words, the error of the plotting board proper should at least not be greater than that caused by the error in producing parallelism. A form of board made of heavy ground glass or other suitable material, on which the plotting is indicated by the intersection of perfectly straight and unbending hair-lines, has been experimented upon in a crude way in the laboratory, and the results justify further trials. Such an arrangement would at once remove from the board the long cumbersome mechanical arms which are often a source of much error.

By referring to the table above, giving the error of setting for parallelism by means of the galvanometer from twenty different observations, it is seen that even with the very rough and home made apparatus which was used in these experiments, the results obtained are within the limits of the requirements for good range finding at present.

The mean error of 2.775 minutes of arc, is equivalent to but 1.39 yards at the distance of a mile, or 7.9 yards at the distance of 10,000 yards or 5.68 miles. The probable error of a single observation is but 2.67 minutes of arc, and the greatest error 8.75 minutes corresponds to about 25 yards in 10,000.

The theoretical discussion of a Wheatstone's bridge for an harmonic impressed electromotive force, has a scientific interest independent of its particular application to the range finder problem.

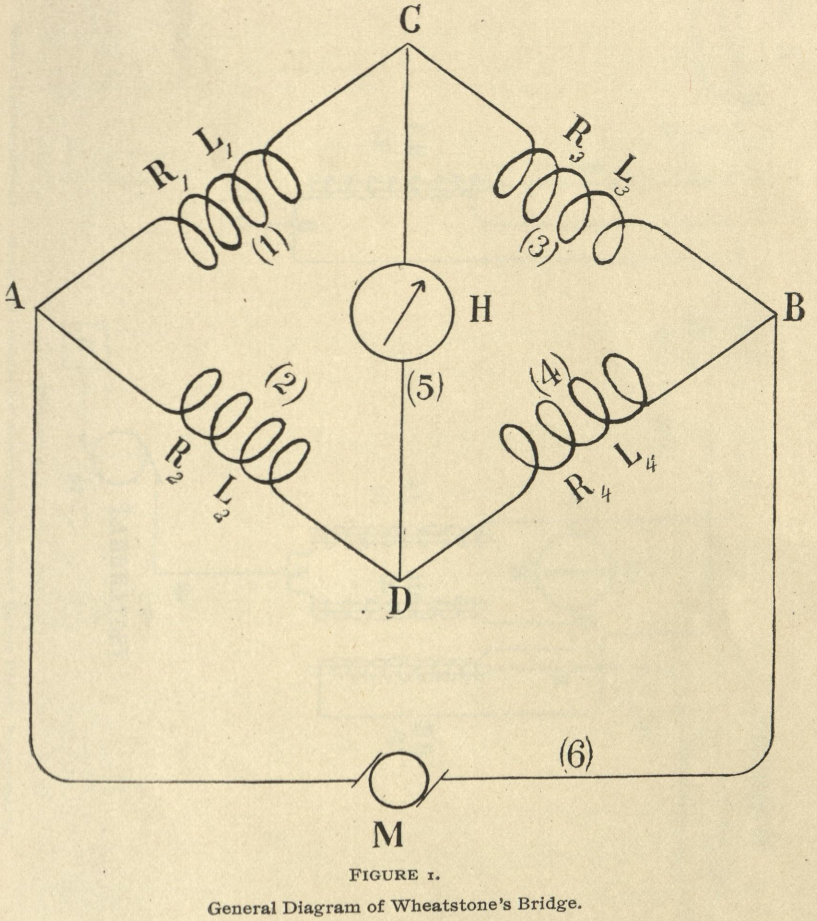

FIGURE 1.

General Diagram of Wheatstone's Bridge.

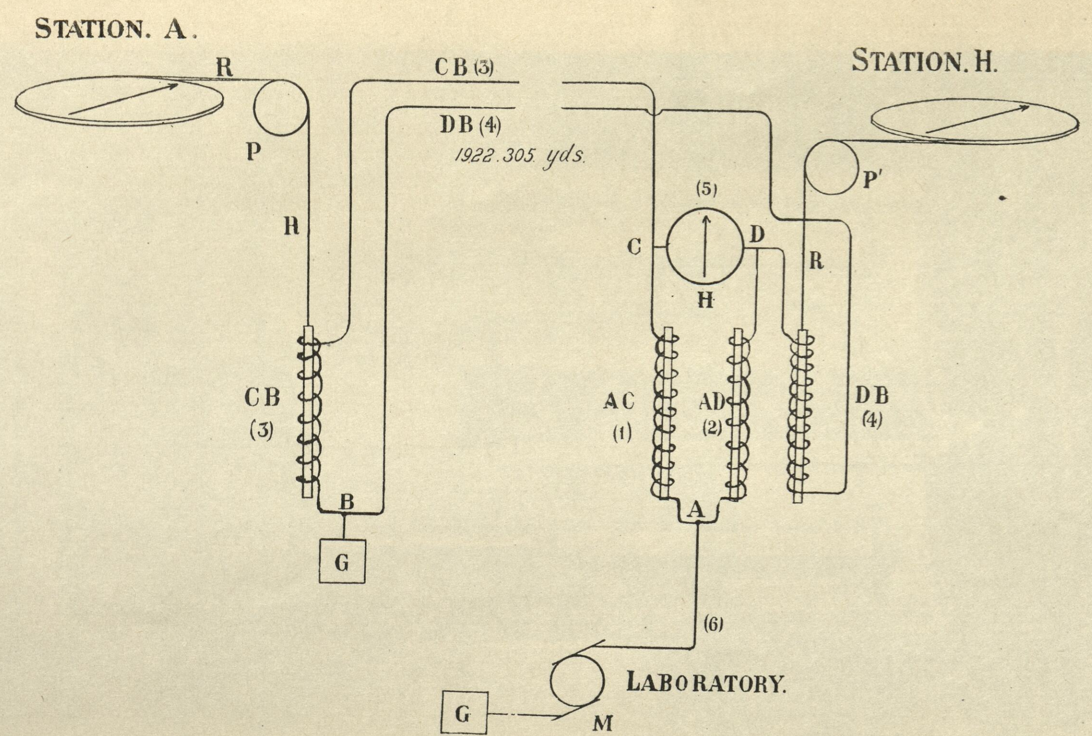

FIGURE 2.

General arrangement of electrical circuits for Alternating Current Range Finder Experiments.

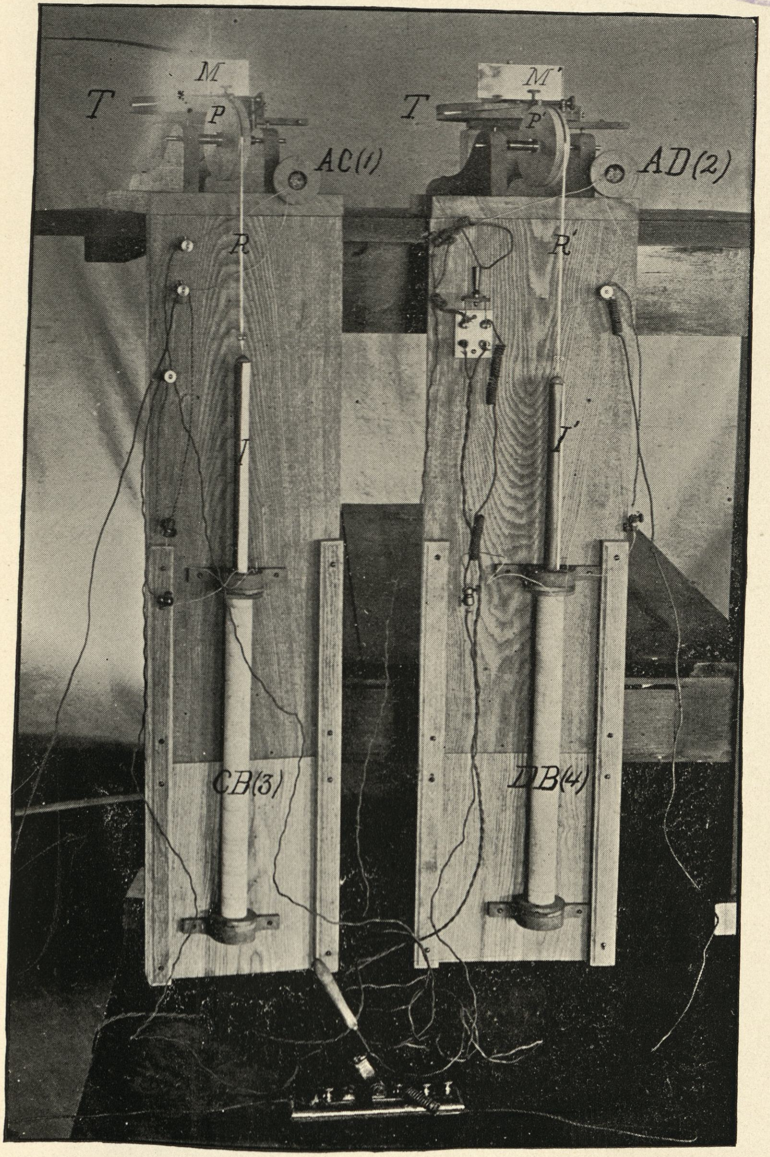

FIGURE 3.

Laboratory Apparatus for testing Alternating Current Range Finder.
Electrical Laboratory, U. S. Artillery School.

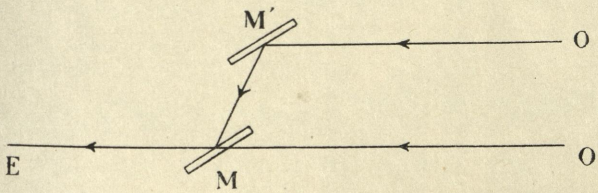

FIGURE 4.

Illustrating method of testing for parallelism similar to the sextant instrument.

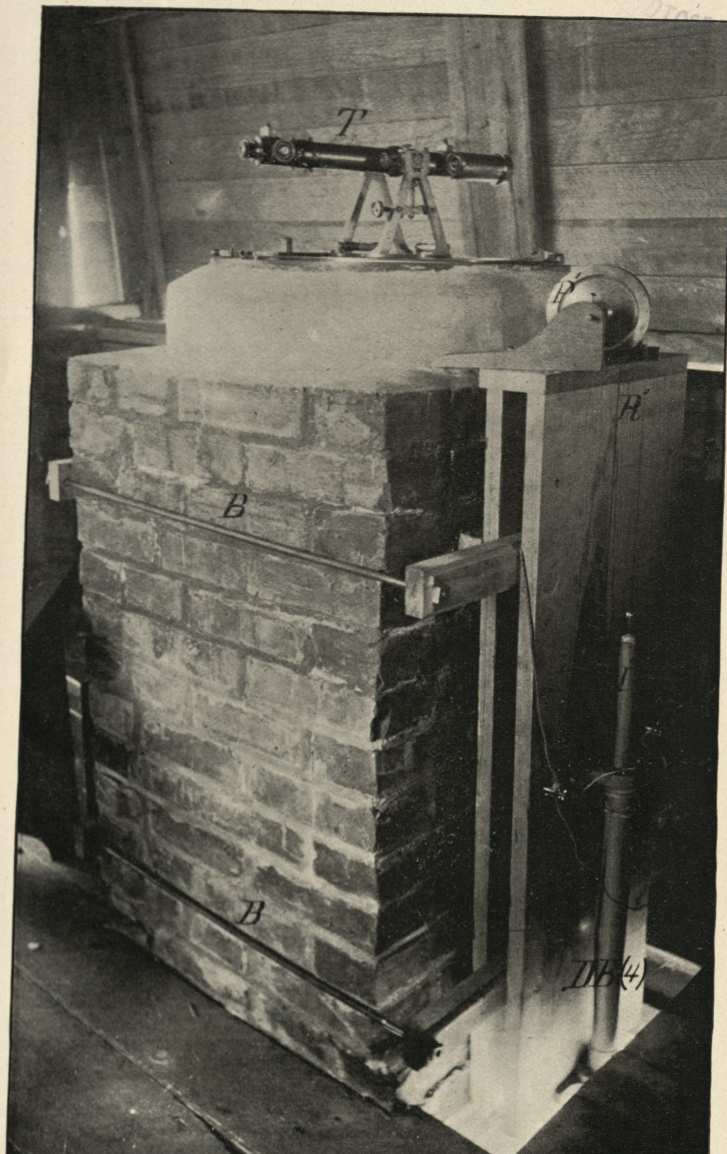

FIGURE 5.

Azimuth Instrument, Station "H", Fort Monroe, Va., showing attachment for Alternating Current Range Finder Experiments.

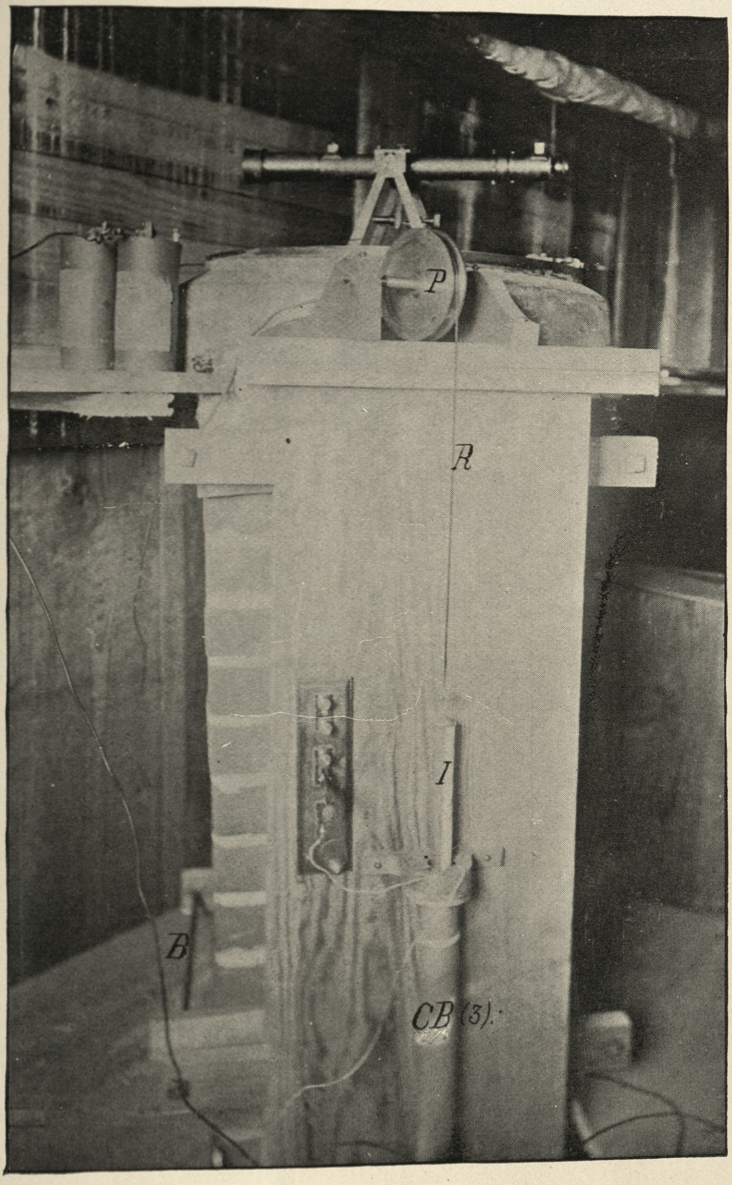

FIGURE 6.

Azimuth Instrument, Station "A", Fort Monroe, Va., showing attachment for Alternating Current Range Finder Experiments.

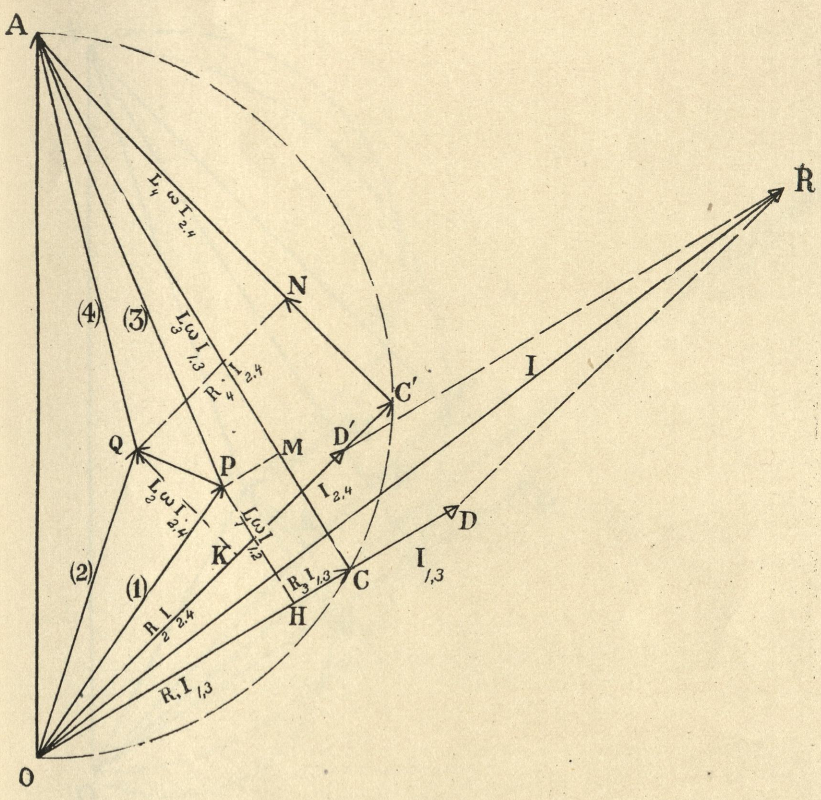

FIGURE 7.

General diagram, illustrating the currents and potential differences in the two branches of a divided circuit, each branch containing two coils with impedance; and showing the condition for zero current in the galvanometer of a Wheatstone's Bridge.

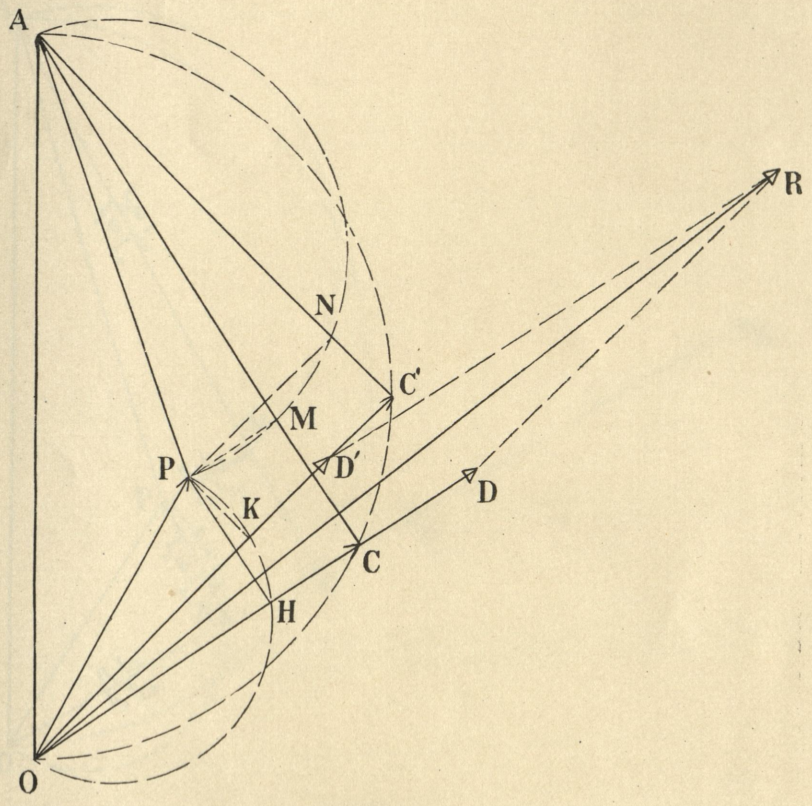

FIGURE 8.

Diagram the same as Fig. 7, except that the points P and Q are brought into coincidence, a necessary condition for zero current in the galvanometer of a Wheatstone's Bridge.

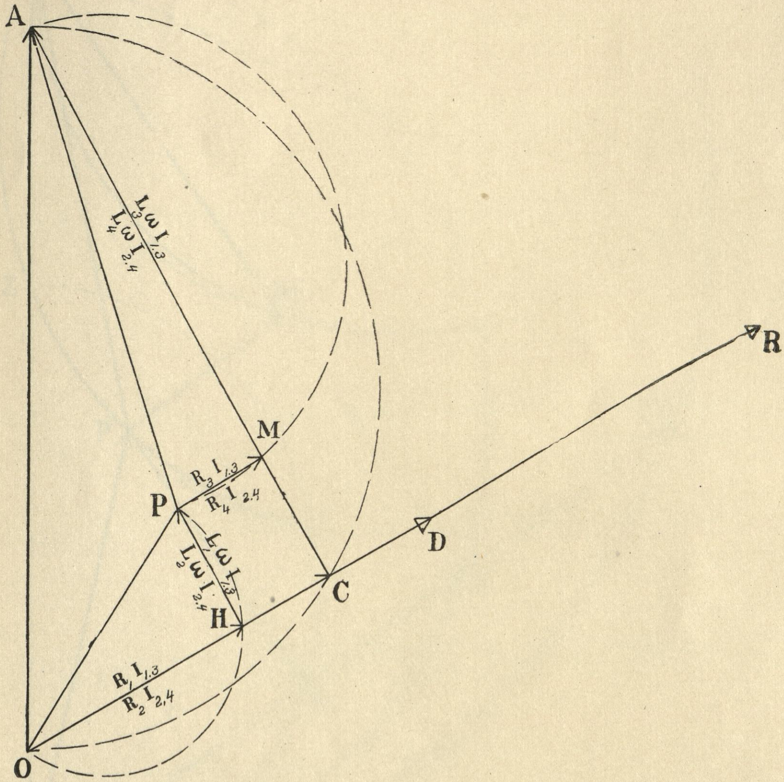

FIGURE 9.

Diagram the same as Fig. 8, except that the points M and N also H and K are brought into coincidence, a condition necessary for zero current in the galvanometer if $R_1 = R_2$ and $L_1 = L_2$.

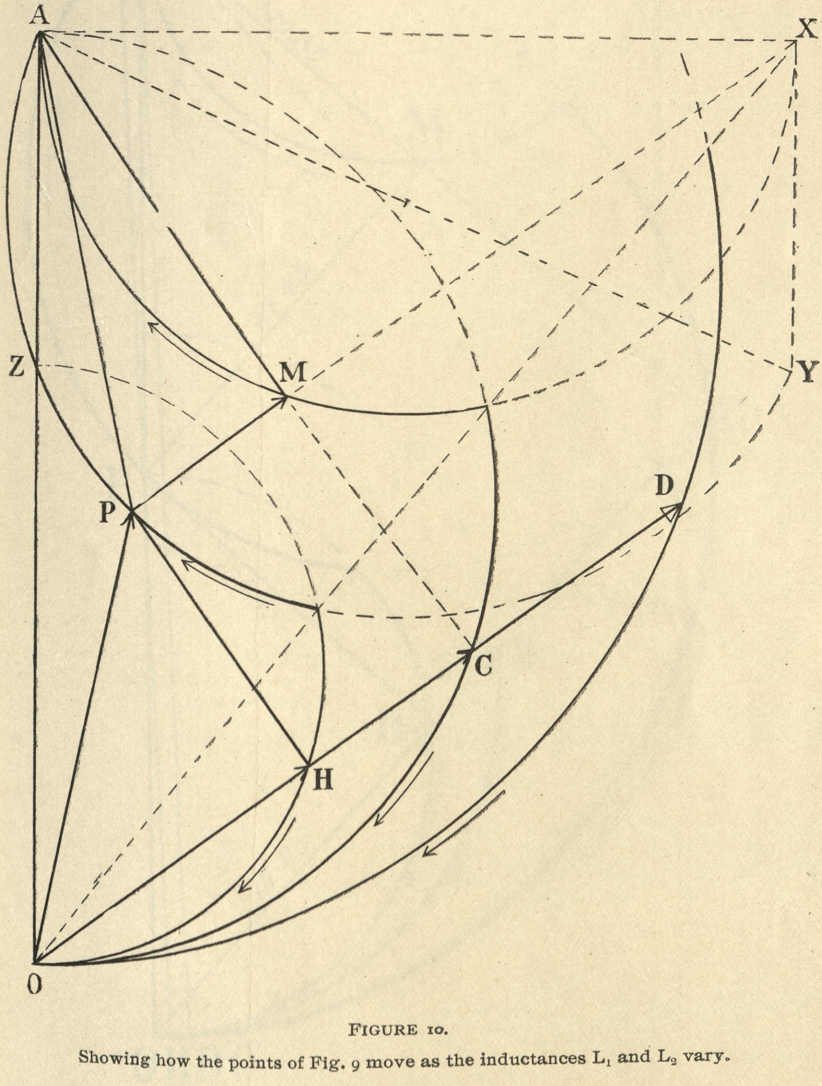

FIGURE 10.

Showing how the points of Fig. 9 move as the inductances L_1 and L_2 vary.

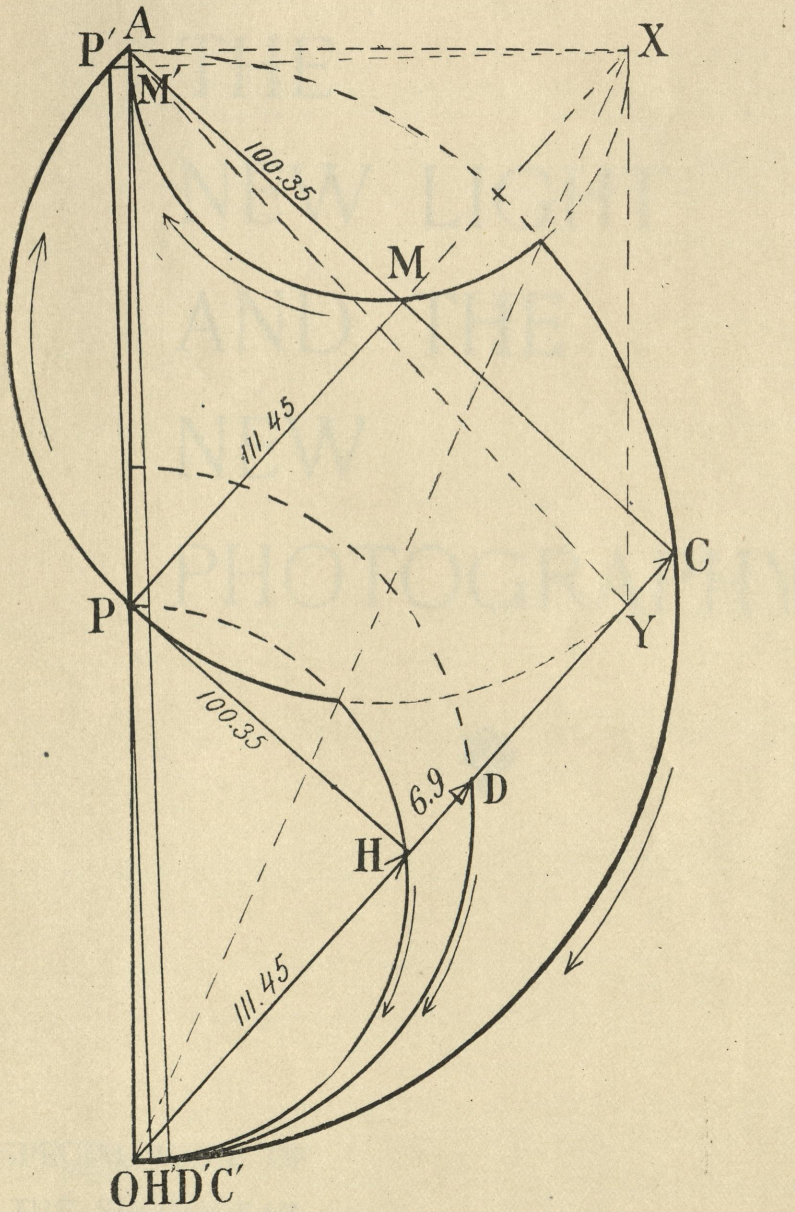

FIGURE 11.

Showing the actual variation of the quantities used from when the iron cores are all out until two of them are entirely in.

Scale: 1 cm. = 20 volts.

" 1 cm. = 1 ampere.